Assignment #2: Rigid Motions and Homogeneous Transformations

Problem 1: In terms of the x_0, y_0, and z_0 coordinates of a fixed frame {0}, the frame {1} has its x_1-axis pointing in the direction (0; 0; 1) and its y_1-axis pointing in the direction (-1; 0; 0), and the frame {2} has its x_2-axis pointing in the direction (1; 0; 0) and its y_2-axis pointing in the direction (0; 0;-1).

(a) Draw by hand the three frames, at different locations so that they are easy to see.

(b) Write down the rotation matrices R_1^0 and R_2^0.

(c) Given R_2^0, how do you calculate R_0^2 without using a matrix inverse?

(d) Write down R_0^2 and verify its correctness using your drawing.

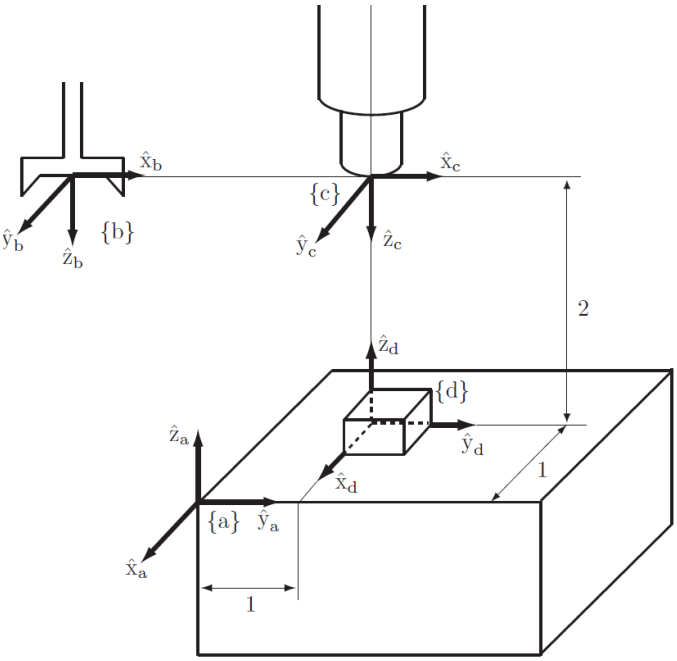
Problem 2: Four reference frames are shown in the robot workspace: the fixed frame {a}, the end-effector frame {b}, the camera frame {c}, and the work-piece frame {d}.

(a) Calculate the following:

- O_d^a , O_c^a , and O_c^d.
- R_d^a , R_c^a , and R_b^a.

(b) Without using the matrix inverse, calculate the following:

- R_a^d , R_a^c , and R_a^b



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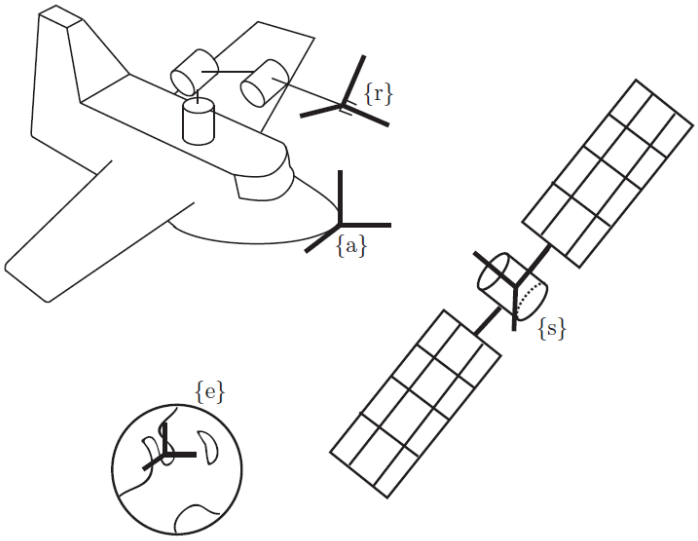
Problem 3: Let p be a point whose coordinates are p = {0.5; 1.2; -3} with respect to the fixed frame {0}. Suppose that p is rotated about the fixed frame x-axis by 30 degrees, then about the fixed-frame y-axis by 135 degrees, and finally about the fixed-frame z-axis by -120 degrees.

(a) Find the rotation matrix **R** that represents the final rotation.
(b) Calculate the new coordinates of the point p with respect to frame {0} after rotation.
(c) Using MATLAB robotics toolbox, verify (a) and (b).

Problem 4:

Consider a robot arm mounted on a spacecraft as shown, in which frames are attached to the Earth {e}, a satellite {s}, the spacecraft {a}, and the robot arm {r}, respectively.



(a) Given T_s^e, T_a^e, and T_r^a find T_s^r.
(b) Suppose that the frame {s} origin as seen from {e} is (1; 1; 1) and that

$$T_r^e = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Write down the coordinates of the frame {s} origin as seen from frame {r}.

Problem 5: Given a fixed frame {0} and a moving frame {1} initially aligned with {0}, perform the following sequence of transformations on {1}:
1. Rotate {1} about the {0} frame x-axis by 30 degrees; call this new frame {2}.
2. Translate {2} along the {0} frame y-axis by 2 units; call this new frame {3}.
3. Rotate {3} about its z-axis by 90 degrees; call this new frame {4}.

(a) What is the final transformation T_4^0 ?
(b) Verify your answer in (a) using MATLAB robotics toolbox with animation.